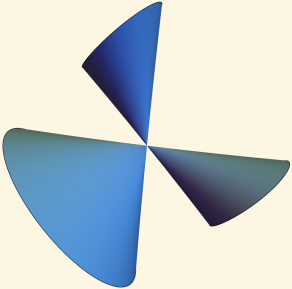

The Donovan–Wemyss conjecture on compound Du Val singularities

An application of the triangulated Auslander–Iyama correspondence

Università degli Studi di Pavia, 24/2/2023

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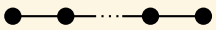
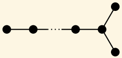
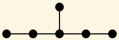
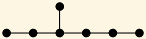
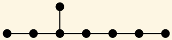
Du Val singularities[1]

They are the isolated surface singularities arising as:

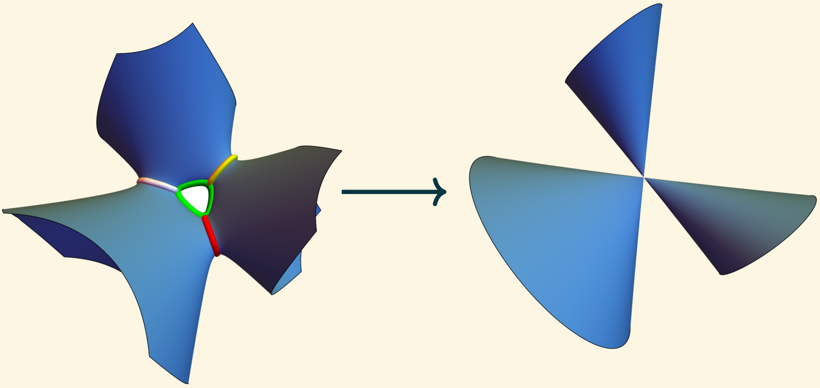
Variety		Functions
$\mathbb{C}^2/G$	$G \subset \mathrm{SL}(2, \mathbb{C})$ finite	$\mathbb{C}[\![x, y]\!]^G$

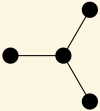
They are classified by *ADE* Dynkin diagrams:

Equation	Diagram
$x^2 + y^2 + z^{n+1}$	 A_n $n \geqslant 1$
$x^2 + y^2 z + z^{n-1}$	 D_n $n \geqslant 4$
$x^2 + y^3 + z^4$	 E_6
$x^2 + y^3 + yz^3$	 E_7
$x^2 + y^3 + z^5$	 E_8

[1] AKA Kleinian singularities.

Du Val singularities





D_4

Compound Du Val singularities

A ***cDV*** singularity is a hypersurface singularity in $\mathbb{C}^4$ whose generic hyperplane section is Du Val.

Equivalently, a cDV is of the form $\mathrm{Spec}(R)$

$$R \cong \frac{\mathbb{C}[\![x, y, z, t]\!]}{(f + t \cdot g)}$$

with $\mathbb{C}[\![x, y, z]\!]/(f)$ Du Val and $g \in \mathbb{C}[\![x, y, z, t]\!]$ arbitrary.

Unlike Du Val singularities, cDV singularities ***are not classified*** in terms of invariants.

From now on, we assume that our cDV singularities are ***isolated***.

Contraction algebras

cDV singularities $X = \mathrm{Spec}(R)$ have ***minimal models*** $f\colon Y \to X$.

wemyss_2018_flops_clusters_homological established bijections:

Geometry	**Algebra**
Minimal model	$T \in D^{\mathrm{sg}}(R)$ basic maximal rigid
Smooth minimal model	$T \in D^{\mathrm{sg}}(R)$ basic $2\mathbb{Z}$-cluster tilting

Here $D^{\mathrm{sg}}(R)$ is the ***singularity category*** of $\mathrm{Spec}(R)$,

$$D^{\mathrm{sg}}(R) = D^{\mathrm{b}}(R)/D^{\mathrm{c}}(R).$$

donovan_wemyss_2016_noncommutative_deformations_flops defined the ***contraction algebra*** of a minimal model as

$$\Lambda = \mathrm{End}_{D^{\mathrm{sg}}(R)}(T).$$

The conjecture

***Conjecture* (donovan_wemyss_2016_noncommutative_deformations_**

Given two isolated cDV singularities R_1 and R_2 with smooth minimal models and associated contraction algebras Λ_1 and Λ_2,

$$R_1 \cong R_2 \iff D(\Lambda_1) \simeq D(\Lambda_2).$$

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- For type D, contraction algebras distinguish between some cDV singularities whose other invariants coincide.

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- For type A, it follows from **reid_1983_minimal_models_canonical**.
- For type D, contraction algebras distinguish between some cDV singularities whose other invariants coincide.
- **august_2020_finiteness_derived_equivalence**: the class of contraction algebras of an isolated cDV singularity form a derived equivalence class. Hence, on the right we can assume

Derived contraction algebras

The ***derived contraction algebra*** of a minimal model of a cDV is

$$\Lambda^{\mathrm{dg}} = \mathbb{R}\mathrm{End}_{D^{\mathrm{sg}}(R)}(T), \qquad \Lambda = H^0(\Lambda^{\mathrm{dg}}), \qquad D^{\mathrm{c}}(\Lambda^{\mathrm{dg}}) \simeq D^{\mathrm{sg}}(R).$$

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Theorem* (hua_keller_2021_cluster_categories_rational*)

Given isolated cDV singularities R_1 y R_2 with smooth minimal models and associated derived contraction algebras Λ_1^{dg} y Λ_2^{dg},

$$\Lambda_1^{\mathrm{dg}} \simeq \Lambda_2^{\mathrm{dg}} \Longrightarrow R_1 \cong R_2.$$

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The proof uses that Λ^{dg} recovers the ***Tyurina algebra***, which classifies hypersurface singularities in a given dimension,

$$HH^0(\Lambda^{\mathrm{dg}}) \cong \frac{\mathbb{C}[\![x,y,z,t]\!]}{\left(h, \frac{\partial h}{\partial x}, \frac{\partial h}{\partial y}, \frac{\partial h}{\partial z}, \frac{\partial h}{\partial t}\right)}, \quad R \cong \frac{\mathbb{C}[\![x,y,z,t]\!]}{(h)}, \quad h = f + t \cdot g.$$

The conjecture

Conjecture (Donovan–Wemyss after August and Hua–Keller)

Dadas cDV aisladas R_1 y R_2 con modelos minimales lisos con álgebras de contracción derivadas Λ_1^{dg} y Λ_2^{dg},

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Question

Is the derived contraction algebra Λ^{dg} determined by its 0-dimensional cohomology $\Lambda = H^0(\Lambda^{\mathrm{dg}})$?

Properties of $D^{sg}(R)$

- **buchweitz_2021_maximal_cohenmacaulay_modules**: since R is complete local

$$D^{sg}(R) \simeq \underline{\mathrm{CM}}(R).$$

- **yoshino_1990_cohenmacaulay_modules_cohenmacaulay**: ***Hom-finite***,

$$\dim \mathrm{Hom}(M, N) < \infty.$$

- **MR570778**: 2-***periodic*** since $\mathrm{Spec}(R)$ is a hypersurface,

$$M \cong M[2].$$

- **MR0480688**: 2-***Calabi Yau*** since $\dim R = 3$,

$$\mathrm{Hom}(M, N)^{\vee} \cong \mathrm{Hom}(N, M[2]).$$

The triangulated Auslander–Iyama correspondence

***Theorem* (muro_2022_enhanced_finite_triangulated, *d*=1, jasso_muro_2022_triangulated_auslander_iyama *in general*)**

Let $d \geqslant 1$ and let k be a perfect field. There are bijective correspondence between:

1. Quasi-isomorphism classes of DG-algebras A such that:
    a. $H^0(A)$ is basic and finite-dimensional.
    b. $A \in D^c(A)$ is $d\mathbb{Z}$-cluster tilting.
2. Equivalence classes of pairs $(\mathcal{T}, c)$ with:
    a. $\mathcal{T}$ a Hom-finite Karoubian algebraic triangulated category.
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3. Isomorphism classes of pairs (Λ, I) where:
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surjective!

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$^0(A)$

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The conjecture

Corollary (Keller 2022)

The Donovan–Wemyss conjecture holds.

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The Donovan–Wemyss conjecture holds.

Proof.

The triangulated Auslander–Iyama correspondence for $d = 2$ associates

$$\Lambda^{\mathrm{dg}} \longmapsto (D^{\mathrm{sg}}(R), T) \longmapsto (\Lambda, I = \Lambda)$$

since, by 2-periodicity, we have equalities of Λ-bimodules

$$I = \mathrm{Hom}(T[2], T) \cong \mathrm{Hom}(T, T) = \Lambda.$$

Hence, the derived contraction algebra Λ^{dg} is determined by the non-derived contraction algebra Λ. ∎

$d\mathbb{Z}$-cluster tilting objects

Given $\mathcal{T}$ Hom-finite, $c \in \mathcal{T}$ is ***d-cluster tilting*** if it is basic and

$$\begin{aligned} \operatorname{add}(c) &= \{x \in \mathcal{T} \mid \mathcal{T}(x, c[i]) = 0 \ \forall 0 < i < d\} \\ &= \{x \in \mathcal{T} \mid \mathcal{T}(c[i], x) = 0 \ \forall 0 < i < d\}. \end{aligned}$$

Moreover, $c \in \mathcal{T}$ is ***$d\mathbb{Z}$-cluster tilting*** if in addition

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Moreover, $c \in \mathcal{T}$ is ***$d\mathbb{Z}$-cluster tilting*** if in addition

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A choice $\phi \colon c[d] \cong c$ gives rise to an algebra automorphism

$$\sigma \colon \Lambda = \mathcal{T}(c, c) \xrightarrow{[d]} \mathcal{T}(c[d], c[d]) \cong \mathcal{T}(c, c) = \Lambda, \qquad \lambda \mapsto \phi(\lambda[d])\phi^{-1},$$

well-defined up to inner algebra automorphisms

$$[\sigma] \in \mathrm{Out}(\Lambda) \cong \mathrm{Pic}(\Lambda).$$

The corresponding ***invertible*** Λ-bimodule is

$$\mathcal{T}(c[d], c) \cong \Lambda_\sigma = (\mathrm{id}_\Lambda \otimes \sigma)^* \Lambda.$$

Properties of the contraction algebra

- Λ is ***symmetric***,

$$\Lambda^* = \mathrm{Hom}(T, T)^* \cong \mathrm{Hom}(T, T[2]) \cong \mathrm{Hom}(T, T) = \Lambda.$$

- In the smooth case, Λ is ***4-periodic***,

$$0 \to \Lambda \to P_4 \to P_3 \to P_2 \to P_1 \to P_0 \to \Lambda \to 0$$

exact with P_i projective Λ-bimodules.

Formalidad

Una álgebra graduada B es ***intrínsecamente formal*** si dadas dos DG-álgebras A_1, A_2,

$$H^*(A_1) \cong H^*(A_2) \cong B \Longrightarrow A_1 \simeq A_2.$$

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***Theorem* (jasso_keller_muro_2023_donovan_wemyss_conjecture)**

Los siguientes enunciados son equivalentes:

1. $\Lambda[u^{\pm 1}]$ es intrínsecamente formal.
2. $\Lambda = \mathbb{C}$.
3. $R = \mathbb{C}[\![x, y, z, t]\!]/(xy - zt)$.
4. $f \colon Y \to X$ es el *flop* de Atiyah.

A_∞-álgebras

Una ***A-algebra*** $(A, m_1, m_2, m_3, \dots)$ es un espacio vectorial graduado A equipado con operaciones de grado $2 - n$

$$m_n \colon A \otimes \overset{n}{\cdots} \otimes A \longrightarrow A,$$

tales que:

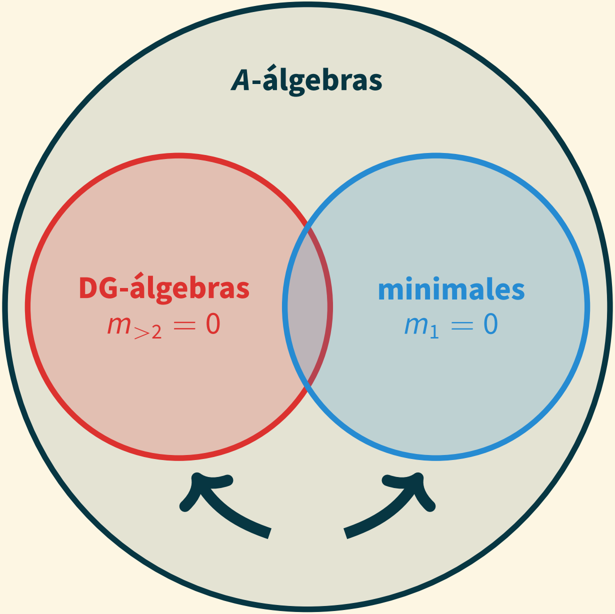
- (A, m_1) es un complejo, $m_1^2 = 0$;
- $ab = m_2(a, b)$ satisface la regla de Leibniz resp. $d = m_1$,

$$d(ab) = d(a)b + (-1)^{|a|} a d(b);$$

- el producto m_2 es asociativo salvo homotopía m_3;
- …

En particular $H^*(A)$ es un álgebra graduada.

A_∞-algebras



Modelos minimales

El ***modelo minimal*** de una DG-álgebra A con $H^*(A)$ concentrada en grado par es

$$(H^*(A), 0, m_2, 0, m_4, 0, m_6, \dots).$$

El ***producto de Massey universal (UMP)*** de A es

$$\{m_4\} \in HH^{4,-2}(H^*(A)).$$

Recordemos que el ***complejo de Hochschild*** está dado por

$$C^n(B) = \mathrm{Hom}(B \otimes \overset{n}{\cdots} \otimes B, B).$$

Álgebras de Massey

Un ***álgebra de Massey*** (B, m) es un álgebra graduada B concentrada en grados pares equipada con

$$m \in HH^{4,-2}(B), \qquad \frac{1}{2}[m, m] = 0.$$

El álgebra de Massey de una DG-álgebra A con $H^*(A)$ concentrada en grados pares es

$$(H^*(A), \{m_4\}).$$

La ***cohomología de Hochschild*** de un álgebra de Massey

$$HH^*(B, m)$$

es la cohomología del complejo

$$(HH^*(B), [m, -]).$$

Formalidad de Massey

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Theorem* (jasso_muro_2022_triangulated_auslander_iyama*)

1. $HH^{n+2,-n}(B, m) = 0, n > 2 \Rightarrow (B, m)$ intrínsecamente formal.

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2. El álgebra de Massey $(\Lambda[u^{\pm 1}], \{m_4^{\Lambda^{\mathrm{dg}}}\})$ de un álgebra de contracción derivada Λ^{dg} satisface **??**.

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2. El álgebra de Massey $(\Lambda[u^{\pm 1}], \{m_4^{\Lambda^{\mathrm{dg}}}\})$ de un álgebra de contracción derivada Λ^{dg} satisface **??**.
3. Los UMP $\{m_4\} \in HH^{4,-2}(\Lambda[u^{\pm 1}])$ asociados a álgebras de contracción derivadas Λ^{dg} forman una $\mathrm{Aut}(\Lambda[u^{\pm 1}])$-órbita.

Bibliografía I