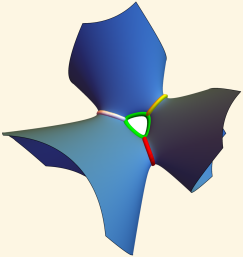

The Donovan–Wemyss conjecture on compound Du Val singularities

An application of the triangulated Auslander–Iyama correspondence

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Contraction algebras

R complete local isolated compound Du Val (cDV) singularity with smooth minimal model.

$\mathcal{T} = D^{\mathrm{sg}}(R) = D^{\mathrm{b}}(R)/\operatorname{perf}(R)$ singularity category.

$$\{\text{Minimal models } Y \to X = \operatorname{Spec}(R)\}$$

(wemyss_2018_flops_clusters_homological) $\uparrow\downarrow$

$$\{2\mathbb{Z}\text{-cluster tilting objects } c \in \mathcal{T}\}$$

(august_2020_finiteness_derived_equivalence) $\uparrow\downarrow$ $\Lambda = \mathcal{T}(c,c)$ ***contraction algebra*** (donovan_wemyss_2016_noncommutati

$$\left\{\begin{array}{l}\text{Derived equivalence class of finite-}\\ \text{dimensional basic algebras } \Lambda\end{array}\right\}$$

The Donovan–Wemyss conjecture

***Conjecture* (donovan_wemyss_2016_noncommutative_deformations, august_2020_finiteness_derived_equivalence)**

Given complete isolated cDV singularities R_1, R_2 with smooth minimal models and contraction algebras Λ_1, Λ_2,

$$R_1 \cong R_2 \iff D(\Lambda_1) \simeq D(\Lambda_2).$$

- $\Rightarrow$ follows from **wemyss_2018_flops_clusters_homological** and **dugas_2015_construction_derived_equivalent**.
- $\Leftarrow$ follows from **hua_keller_2021_cluster_categories_rational** and **jasso_muro_2022_triangulated_auslander_iyama**, as noticed by Keller.

By **august_2020_finiteness_derived_equivalence**, in $\Leftarrow$ we can assume $\Lambda_1 \cong \Lambda_2$.

$2\mathbb{Z}$-derived contraction algebras

Take the derived endomorphism DG algebra of $c \in \mathcal{T}$ instead.
We call it ***$2\mathbb{Z}$-derived contraction algebra***,

$$\Lambda^{\mathrm{dg}} = \mathbb{R}\mathcal{T}(c, c), \qquad \Lambda = H^0(\Lambda^{\mathrm{dg}}), \qquad \mathcal{T} = \mathrm{perf}(\Lambda^{\mathrm{dg}}).$$

***Theorem* (hua_keller_2021_cluster_categories_rational)**

Given an isolated cDV singularity

$$R = \frac{\mathbb{C}[\![u, v, x, y]\!]}{(f)}$$

with smooth minimal model

$$HH^0(\Lambda^{\mathrm{dg}}, \Lambda^{\mathrm{dg}}) = \frac{\mathbb{C}[\![u, v, x, y]\!]}{\left(f, \frac{\partial f}{\partial u}, \frac{\partial f}{\partial v}, \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)}$$ ***Tyurina algebra***.

$2\mathbb{Z}$-derived contraction algebras

***Corollary* (hua_keller_2021_cluster_categories_rational)**

Given complete local isolated cDV R_1, R_2 with smooth minimal models and $2\mathbb{Z}$-derived contraction algebras $\Lambda_1^{\mathrm{dg}}, \Lambda_2^{\mathrm{dg}}$,

$$\begin{aligned}\Lambda_1^{\mathrm{dg}} \simeq \Lambda_2^{\mathrm{dg}} &\Rightarrow R_1, R_2 \text{ have the same Tyurina algebra}\\ &\Rightarrow R_1 \cong R_2 \text{ (}\textbf{mather\_yau\_1982\_classification\_isolated\_hyper}\end{aligned}$$

What remains to be proved

We start with results from **jasso_muro_2022_triangulated_auslander_iyama**.

Theorem

Given complete local isolated cDV R_1, R_2 with smooth minimal models, contraction algebras Λ_1, Λ_2 and $2\mathbb{Z}$-derived contraction algebras $\Lambda_1^{dg}, \Lambda_2^{dg}$,

$$\Lambda_1 \cong \Lambda_2 \Rightarrow \Lambda_1^{dg} \simeq \Lambda_2^{dg}.$$

In order to prove this theorem we must answer the following question.

Question

Can we recover Λ^{dg} from $\Lambda = H^0(\Lambda)$?

The cohomology of Λ^{dg}

- Recall that $\Lambda^{\mathrm{dg}} = \mathbb{R}\mathcal{T}(c, c)$.

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Question

Can we recover Λ^{dg} from $H^*(\Lambda) = \Lambda[t^{\pm 1}]$?

Formality

A DGA A is ***formal*** if $A \simeq H^*(A)$.

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***Theorem* (kadeishvili_1988_structure_infty_algebra)**

$HH^{n,2-n}(B,B) = 0$, $n > 2 \Rightarrow B$ is intrinsically formal.

Theorem

Given a complete local isolated cDV singularity R with contraction algebra Λ, TFAE:

1. $\Lambda[t^{\pm 1}]$ is intrinsically formal .
2. $\Lambda = \mathbb{C}$.
3. $R = \mathbb{C}[[u, v, x, y]]/(uv - xy)$.
4. $f\colon Y \to X$ is the Atiyah flop.

A_∞-algebras

An ***A_∞-algebra*** $(A, m_1, m_2, m_3, \dots)$ is a graded vector space A equipped with operations

$$m_n : A \otimes \overset{n}{\cdots} \otimes A \longrightarrow A, \qquad |m_n| = 2 - n, \qquad n \geq 1,$$

satisfying some equations:

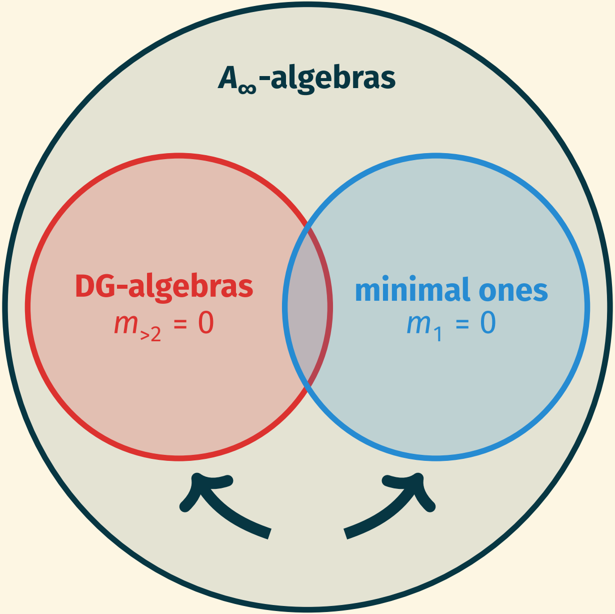
- (A, m_1) is a complex, $m_1^2 = 0$;
- $xy = m_2(x, y)$ satisfies the Leibniz rule w.r.t. $\partial = m_1$,

$$\partial(xy) = \partial(x)y + (-1)^{|x|} x\partial(y);$$

- the product m_2 is associative up to the homotopy m_3;
- ...

In particular $H^*(A)$ is a graded algebra.

A_∞-algebras



Minimal A_∞-models

A ***minimal*** ***A_∞-model*** of Λ^{dg} looks like

$$(\Lambda[t^{\pm 1}], m_4, m_6, m_8, \dots).$$

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Example (The pagoda)

Consider

$$R = \frac{\mathbb{C}[\![u,v,x,y]\!]}{(uv - (x - y^2)(x + y^2))}, \qquad \Lambda = \frac{\mathbb{C}[y]}{(y^2)},$$

$$\Lambda^{\mathrm{dg}} = \mathbb{C}[y]\langle w^{\pm 1}\rangle, \quad |y| = 0, \quad |w| = -1, \quad d(y) = 0, \quad d(w) = y^2,$$

and for $n \geq 4$ the only non-trivial m_n is

$$\Lambda[t^{\pm 1}] = \frac{\mathbb{C}[y, t^{\pm 1}]}{(y^2)}, \qquad m_4(yt^a, yt^b, yt^c, yt^d) = t^{a+b+c+d+1}.$$

Hochschild cohomology

The ***Hochschild cohomology*** of a graded algebra B with coefficients on an B-bimodule M is

$$HH^{\bullet,*}(B,M) = \mathrm{Ext}^{\bullet,*}_{B^e}(B,M).$$

$\bullet$ = ***Hochschild degree*** = extension length.
$*$ = ***inner degree***, coming from the fact that A is graded.

It is the cohomology of the ***Hochschild complex***

$$C^n(B,M) = \mathrm{Hom}_{\mathbb{C}}(B\otimes \overset{n}{\cdots} \otimes B, M).$$

The A_∞-operations of $(\Lambda[t^{\pm 1}], m_4, m_6, m_8, \dots)$ are Hochschild cochains

$$m_n \in C^{n,2-n}(\Lambda[t^{\pm 1}], \Lambda[t^{\pm 1}]).$$

Universal Massey products

The ***universal Massey product (UMP)*** of Λ^{dg} is

$$\{m_4\} \in HH^{4,-2}(\Lambda[t^{\pm 1}], \Lambda[t^{\pm 1}]).$$

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If $j : \Lambda \hookrightarrow \Lambda[t^{\pm 1}]$ is the degree 0 inclusion, the ***restriected UMP*** is

$$j^*\{m_4\} \in HH^{4,-2}(\Lambda, \Lambda[t^{\pm 1}]) = HH^4(\Lambda, \Lambda \cdot t) = \mathrm{Ext}^4_{\Lambda^e}(\Lambda, \Lambda).$$

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Theorem

The restricted UMP $j^*\{m_4\}$ can be represented by an extension

$$\Lambda \hookrightarrow P_4 \to P_3 \to P_2 \to P_1 \twoheadrightarrow \Lambda$$

with P_i projective Λ-bimodules $i = 1, 2, 3, 4$.

This is connected to ***4-angulated categories*** in the sense of **geiss_keller_oppermann_2013_angulated_categories**.

Universal Massey products

Example (The pagoda)

In this case

$$\Lambda = \frac{\mathbb{C}[y]}{(y^2)}, \qquad \mathrm{Ext}^{\bullet}_{\Lambda^e}(\Lambda,\Lambda) = \frac{\mathbb{C}[y,\varphi,\psi]}{(y^2, y\varphi, y\psi)}, \quad |\varphi| = 1, \ |\psi| = 2,$$

where $\varphi(y) = y$, $\psi(y,y) = 1$. Moreover, ψ is represented by

$$0 \to \Lambda \xrightarrow{1\otimes y + y\otimes 1} \Lambda^e \xrightarrow{1\otimes y - y\otimes 1} \Lambda^e \xrightarrow{\text{product}} \Lambda \to 0$$

and $j^*\{m_4\} = \psi^2$ is obtained by splicing this extension with itself.

Hochschild–Tate cohomology

Since Λ is self-injective, we can define its ***Hochschild–Tate cohomology*** with coefficients in a Λ-bimodule M

$$\underline{HH}^{\bullet}(\Lambda, M) = \underline{\mathrm{Ext}}^{\bullet}_{\Lambda^e}(\Lambda, M)$$

from a periodic resolution of Λ as a Λ-bimodule.

$$HH^{>0}(\Lambda, M) = \underline{HH}^{>0}(\Lambda, M).$$

The previous theorem is equivalent to:

Theorem

The rUMP $j^*\{m_4\}$ is a bidegree $(4, -2)$ unit in $\underline{HH}^{\bullet,*}(\Lambda, \Lambda[t^{\pm 1}])$.

Hochschild–Tate cohomology

Example (The pagoda)

In this case

$$\Lambda = \frac{\mathbb{C}[y]}{(y^2)}, \qquad \underline{HH}^{\bullet,*}(\Lambda, \Lambda[t^{\pm 1}]) = \mathbb{C}[\varphi, \psi^{\pm 1}, t^{\pm 1}],$$

with

$$|\varphi| = (1, 0), \qquad |\psi| = (2, 0), \qquad |t| = (0, -2),$$

and $j^*\{m_4\} = \psi^2 t$ is obviously a unit.

Hochschild–Tate cohomology

Corollary

TFAE:

1. Λ^{dg} is formal.
2. $\{m_4\} = 0$.
3. $\underline{HH}^{\bullet,*}(\Lambda, \Lambda[t^{\pm 1}])$ has a trivial unit.
4. $\underline{HH}^{\bullet,*}(\Lambda, \Lambda[t^{\pm 1}]) = 0$.
5. The ***stable center*** $\underline{Z}(\Lambda) = Z(\Lambda)/\{\Lambda \to \Lambda^e \to \Lambda\} = 0$.
6. Λ is semisimple.
7. $\Lambda = \mathbb{C}$.
8. $\Lambda[t^{\pm 1}]$ is intrinsically formal.

Intrinsical formal-ish-ty

A graded algebra B is ***instrinsically formal*** if, given DGAs A_1, A_2

$$H^*(A_1) = H^*(A_2) = B \quad \Longrightarrow \quad A_1 \simeq A_2.$$

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A ***Massey algebra*** (B, m) consists of a graded algebra $B = B^{\text{even}}$

$$m \in HH^{4,-2}(B, B), \qquad \frac{1}{2}[m, m] = 0.$$

The Massey algebra of Λ^{dg} is

$$(\Lambda[t^{\pm 1}], \{m_4\}),$$

and similarly for DGAs A with $H^*(A) = H^{\text{even}}(A)$.

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A Massey algebra (B, m) is ***instrinsically formal*** if, given A_1, A_2

$$H^*(A_1) = H^*(A_2) = B, \;\; \{m_4^{A_1}\} = \{m_4^{A_2}\} = m \quad \Longrightarrow \quad A_1 \simeq A_2.$$

Intrinsical formal-ish-ty

The ***Hochschild complex*** of a Massey algebra (B, m) is

$$C^{\bullet,*}(B, m) = HH^{\bullet,*}(B, B), \qquad \partial = [m, -].$$

The ***Hochschild cohomology*** is denoted by

$$HH^{\bullet,*}(B, m).$$

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Theorem

$HH^{n,2-n}(B, m) = 0$, $n > 4 \Rightarrow (B, m)$ is intrinsically formal.

Related to the A_∞-obstruction theory of **muro_2020_enhanced_obstruction_theory**.

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Proposition

$HH^{n,t}(\Lambda[t^{\pm 1}], \{m_4\}) = 0$ for $n > 4$ and $t \in \mathbb{Z}$.

It follows from $j^*\{m_4\}$ being a Hochschild–Tate unit.

Intrinsical formal-ish-ty

Example (The pagoda)

In this case

$$\Lambda = \frac{\mathbb{C}[y]}{(y^2)}, \qquad HH^{\bullet,*}(\Lambda[t^{\pm 1}], \Lambda[t^{\pm 1}]) = \frac{\mathbb{C}[y, \varphi, \psi, t^{\pm 1}, \delta]}{(y^2, y\varphi, y\psi)},$$

$$|\varphi| = (1,0), \qquad |\psi| = (2,0), \qquad |t| = (0,-2), \qquad |\delta| = (1,0),$$

where $\delta(t) = -t$ and $\delta(\Lambda) = 0$. The UMP is $\{m_4\} = \psi^2 t$, the differential $[\psi^2 t, -]$ vanishes on generators except for

$$[\psi^2 t, \delta] = \psi^2 t.$$

In Hochschild degrees > 4, there is a null-homotopy

$$x \mapsto \delta\psi^{-2}t^{-1}x.$$

The triangulated Auslander–Iyama correspondence

Theorem

Let $d \geq 1$ and let k be a ***perfect field***. There are bijective correspondence between:

1. Quasi-isomorphism classes of DG-algebras A such that:
   a. $H^0(A)$ is basic and finite-dimensional.
   b. $A \in \operatorname{perf}(A)$ is $d\mathbb{Z}$-cluster tilting.
2. Equivalence classes of pairs $(\mathcal{T}, c)$ with:
   a. $\mathcal{T}$ a Hom-finite Karoubian algebraic triangulated category.
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$\Lambda = H^0(A)$

$I = H^{-d}(A)$

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The End

References I