Geometry of singularities and homotopy theory

New Perspectives on Stable Homotopy and Beyond

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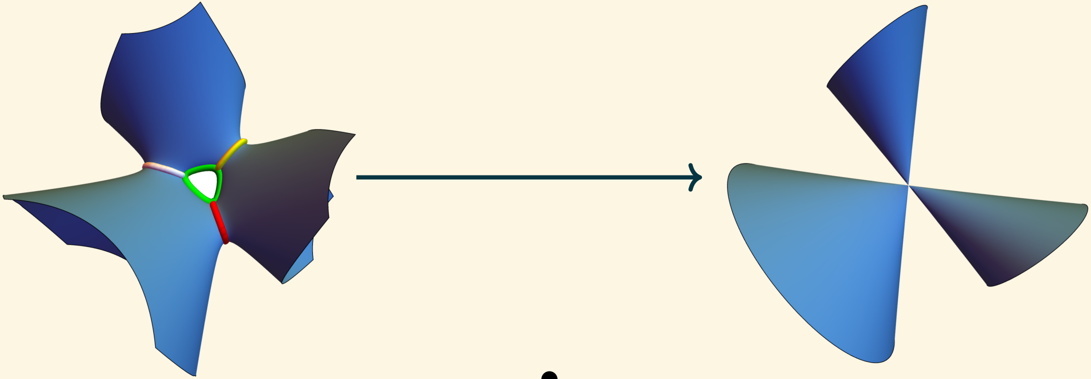
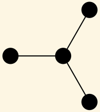
Du Val singularities[1]





D_4

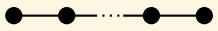
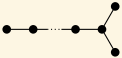
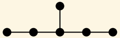
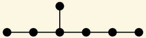
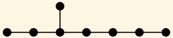
[1] Also known as Kleinian singularities or simple surface singularities.

Du Val singularities

They are isolated surface singularities arising as:

Variety		**Functions**
$\mathbb{C}^2/G$	$G \subset \mathrm{SL}(2, \mathbb{C})$ finite	$\mathbb{C}[\![x, y]\!]^G$

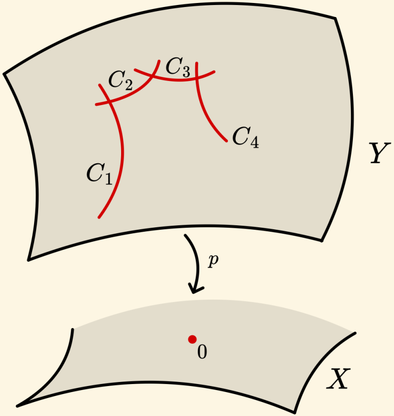
They are classified by *ADE* Dynkin diagrams:

Equation in $\mathbb{C}^3$	**Diagram**	
$x^2 + y^2 + z^{n+1}$		$A_n, n \geq 1$
$x^2 + y^2 z + z^{n-1}$		$D_n, n \geq 4$
$x^2 + y^3 + z^4$		E_6
$x^2 + y^3 + yz^3$		E_7
$x^2 + y^3 + z^5$		E_8

Compound Du Val singularities (Reid 1983)

A 3-dimensional analogue of Du Val singularities:

- X complete, local,
  isolated hypersurface singularity at $0 \in \mathbb{C}^4$.
- The generic hyperplane
  section of X is a Du Val singularity.
- $p : Y \to X$ a crepant resolution, which
  is an isomorphism outside $p^{-1}(0) = \bigcup_{i=1}^{n} C_i$
  with $C_i \cong \mathbb{P}^1$ and Y smooth.



GOAL: Classify
these *cDV singularities* in terms of invariants.

The singularity category

$D^{sg}(X) = D^b(X)/D^c(X)$ is the ***singularity category***; it is an idempotent complete Hom-finite triangulated category with $\Sigma^2 = \mathrm{id}$.

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An object $c \in D^{sg}(X)$ is ***cluster-tilting*** if:

- any object x fits in an exact triangle

$$\underbrace{p_1 \longrightarrow p_0}_{\in \operatorname{add}(c)} \longrightarrow x \longrightarrow \Sigma p_1,$$

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Theorem (August 2020)

For X a cDV, there is a bijection between equivalence classes of:

1. Crepant resolutions $Y \to X$.
2. Basic cluster-tilting objects $c \in D^{sg}(X)$.

Contraction algebras and the Donovan–Wemyss conjecture

A ***contraction algebra*** of a cDV X is the endomorphism algebra $\Lambda = \mathrm{End}(c)$ of a basic cluster-tilting object $c \in D^{sg}(X)$.

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Conjecture (Donovan and Wemyss 2016)
Given two cDVs X, X' with crepant resolutions $Y \to X$, $Y' \to X'$ and contraction algebras Λ, Λ',

$$D(\Lambda) \simeq D(\Lambda') \iff X \cong X'.$$

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Theorem (Hua and Keller 2024)

Given two cDVs X, X' with crepant resolutions $Y \to X$, $Y' \to X'$ and derived contraction algebras $\Lambda_{dg}, \Lambda'_{dg}$,

$$D_{dg}(\Lambda_{dg}) \simeq D_{dg}(\Lambda'_{dg}) \Longleftrightarrow X \cong X'.$$

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"Various enhanced versions of the conjecture are known to be true, but indeed the point of the conjecture is that these enhanced structures are not necessary." — *Wemyss 2023.*

Example (Donovan and Wemyss 2016; Booth 2019)

$$X = \operatorname{Spec}\left(\frac{\mathbb{C}[\![x,y,u,v]\!]}{(u^2+v^2y-x^3-xy^3)}\right)$$ **Laufer flop**;

$$\Lambda = \frac{\mathbb{C}\langle x,y\rangle}{(xy+yx\,,\,x^2-y^3)}$$ **quantum cusp**;

$$t^{\leq 0}\Lambda_{dg} = \mathbb{C}\langle \underbrace{x, y}_{0}, \underbrace{\zeta, \xi}_{-1}, \underbrace{\rho}_{-2} \rangle,$$

$$d(\zeta) = xy+yx, \quad d(\xi) = x^2-y^3, \quad d(\rho) = [\xi, y]+[\zeta, x].$$

Generic hyperplane section

Formality of derived contraction algebras

The cohomology of a derived contraction algebra $\Lambda_{dg} = \mathbb{R}\mathrm{End}(c)$ is

$$H^0\Lambda_{dg} = \mathrm{End}(c) = \Lambda, \qquad H^*\Lambda_{dg} = \bigoplus_{n\in\mathbb{Z}} \mathrm{Hom}(c, \Sigma^n c) = \Lambda[t^{\pm 1}], \quad |t| = -2.$$

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Theorem (Jasso, Keller, and Muro 2024)

Given a cDV X, TFAE:

1. Λ_{dg} is formal.
2. $\Lambda \cong \mathbb{C}$.
3. $X \cong \mathrm{Spec}\,(\mathbb{C}[\![u, v, x, y]\!]/(uv - xy))$ is the **Atiyah flop**.

Universal Massey products

A DGA $\rightsquigarrow (H^*A, m_3, m_4, \dots, m_n, \dots)$ A_∞-algebra **minimal model**,

$$m_n \colon H^*A \otimes \overset{n}{\cdots} \otimes H^*A \longrightarrow H^*A, \qquad |m_n| = 2 - n.$$

[2] Baues and Dreckmann 1989; Benson, Krause, and Schwede 2004; Kaledin 2007; Sagave 2008…

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The ***universal Massey product***[2] **(UMP)** of a DGA A with $H^{\mathrm{odd}}A = 0$

$$\{m_4\} \in \mathrm{HH}^{4,-2}(H^*A, H^*A), \qquad [\{m_4\}, \{m_4\}] = 0.$$

$$m_4(a_1, a_2, a_3, a_4) \in \langle [a_1], [a_2], [a_3], [a_4] \rangle \quad \textbf{Massey product}.$$

The UMP is an obstruction to formality, i.e. if A is formal then $\{m_4\} = 0$.

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Proposition (Jasso, Keller, and Muro 2024)

The Hochschild cohomology of $\Lambda[t^{\pm 1}]$ is the **Gerstenhaber algebra**

$$HH^{\star,*}(\Lambda[t^{\pm 1}], \Lambda[t^{\pm 1}]) \cong HH^{\star}(\Lambda, \Lambda)[t^{\pm 1}, \delta], \quad \delta \ \textbf{Euler class}, \quad |\delta| = (1, 0),$$

$$[t, HH^{\star}(\Lambda, \Lambda)] = 0, \quad [\delta, HH^{\star}(\Lambda, \Lambda)] = 0, \quad [t, t] = 0, \quad [\delta, t] = -t,$$

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and the UMP of Λ_{dg} is

$$\{m_4\} = \left(u + \frac{1}{2}u^{-1}[u, u]\delta\right) t$$

with $u \in HH^4(\Lambda, \Lambda)$ a unit in **Hochschild–Tate cohomology** $\underline{HH}^{\star}(\Lambda, \Lambda)$.

Therefore, $\{m_4\} = 0$ if and only if $\Lambda = \mathbb{C}$.

4-angulated categories (Geiss, Keller, and Oppermann 2013)

The class $u \in HH^4(\Lambda, \Lambda)$ is the first **Postnikov invariant** of $t^{\leq 0}\Lambda_{dg}$. It is a Hochschild–Tate unit because $\mathrm{proj}(\Lambda) \simeq \mathrm{add}(c) \subset D^{sg}(X)$ is a ***4-angulated category*** with shift $\Sigma^2 = \mathrm{id}$ and

$$p_4 \xrightarrow{f_4} p_3 \xrightarrow{f_3} p_2 \xrightarrow{f_2} p_1 \xrightarrow{f_1} \Sigma^2 p_4, \qquad \langle f_1, f_2, f_3, f_4 \rangle \ni 1, \qquad \textit{\textbf{4-angles}}.$$

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In order to complete $f_4 : p_4 \to p_3$ to a 4-angle, we complete it to a triangle in $D^{sg}(X)$

$$p_4 \xrightarrow{f_4} p_3 \xrightarrow{i} x \xrightarrow{q} \Sigma p_4,$$

then, using the cluster-tilting property, we choose an exact triangle in $D^{sg}(X)$,

$$p_2 \xrightarrow{f_2} p_1 \xrightarrow{i'} \Sigma x \xrightarrow{q'} \Sigma p_2, \qquad \text{we rotate it} \qquad x \xrightarrow{-\Sigma^{-1}q'} p_2 \xrightarrow{f_2} p_1 \xrightarrow{i'} \Sigma x,$$

and take

$$\begin{tikzcd}
& p_3 \arrow[rr, "f_3"] \arrow[dr, dashed, "i"] & & p_2 \arrow[dr, "f_2"] & \\
p_4 \arrow[ur, "f_4"] & & x \arrow[ll, dashed, "+1", "q"'] \arrow[ur, dashed, "-\Sigma^{-1}q'"] & & p_1 \arrow[ll, dashed, "+1", "i'"'] \arrow[llll, bend left, "+2\quad f_1"]
\end{tikzcd}$$

4-angulated categories (Geiss, Keller, and Oppermann 2013)

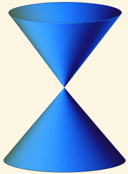
In the example

$$X = \operatorname{Spec}\left(\frac{\mathbb{C}[[x,y,u,v]]}{(uv - x(y^2 - x))}\right), \qquad \Lambda = \mathbb{C}[\varepsilon]/(\varepsilon^2),$$

the only non-trivial indecomposable 4-angle in $\operatorname{proj}(\mathbb{C}[\varepsilon]/(\varepsilon^2))$ is

$$\mathbb{C}[\varepsilon]/(\varepsilon^2) \xrightarrow{\varepsilon} \mathbb{C}[\varepsilon]/(\varepsilon^2) \xrightarrow{\varepsilon} \mathbb{C}[\varepsilon]/(\varepsilon^2) \xrightarrow{\varepsilon} \mathbb{C}[\varepsilon]/(\varepsilon^2) \xrightarrow{\varepsilon} \mathbb{C}[\varepsilon]/(\varepsilon^2),$$

compare Muro, Schwede, and Strickland 2007.



Generic hyperplane section

A generalization of Kadeishvili's formality theorem

A ***Massey algebra*** (H, m) is a graded algebra H with $H^{\mathrm{odd}} = 0$ and

$$m \in \mathrm{HH}^{4,-2}(H, H), \qquad [m, m] = 0.$$

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Theorem (Jasso and Muro 2023)

If A is a DGA with $H^{\mathrm{odd}}A = 0$ and

$$\mathrm{HH}^{n+2,-n}(H^*A, \{m_4\}) = 0, \qquad n > 2,$$

then any other DGA B with $(H^*B, \{m_4^B\}) \cong (H^*A, \{m_4^A\})$ satisfies $B \simeq A$.

The Hochschild cohomology of the contraction Massey algebra

Proposition

The Hochschild cohomology of the Massey algebra of Λ_{dg} is $HH^{n,*}(\Lambda[t^{\pm 1}], \{m_4\}) = 0$ for $n > 4$.

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Proof.

The Hochschild complex

$$(HH^{\star}(\Lambda, \Lambda)[t^{\pm 1}, \delta], d = [\{m_4\}, -])$$

of the Massey algebra has an endomorphism f of bidegree $(4, -2)$ with a nullhomotopy h of bidegree $(1, 0)$,

$$f(x) = \{m_4\} \cdot x, \qquad h(x) = \delta \cdot x, \qquad f = dh + hd,$$

$$\{m_4\} \cdot x = [\{m_4\}, \delta \cdot x] + \delta \cdot [\{m_4\}, x] \quad \textbf{Gerstenhaber relation}.$$

The map f is an isomorphism on horizontal degrees ≥ 2, hence we get the vanishing. ∎

Proof of the Donovan–Wemyss conjecture

Conjecture (Donovan and Wemyss 2016)

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Proof.

$D(\Lambda) \simeq D(\Lambda') \Rightarrow \Lambda \cong \Lambda'$ by changing a resolution (August 2020),

$\blacksquare$

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$$\begin{aligned} D(\Lambda) \simeq D(\Lambda') &\Rightarrow \Lambda \cong \Lambda' && \text{by changing a resolution (August 2020),} \\ &\Rightarrow (\Lambda[t^{\pm 1}], \{m_4\}) \cong (\Lambda[t^{\pm 1}], \{m_4'\}) && \text{by the UMP computation,} \end{aligned}$$

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&\Rightarrow \Lambda_{dg} \simeq \Lambda'_{dg} && \text{by the generalized Kadeishvili theorem,}
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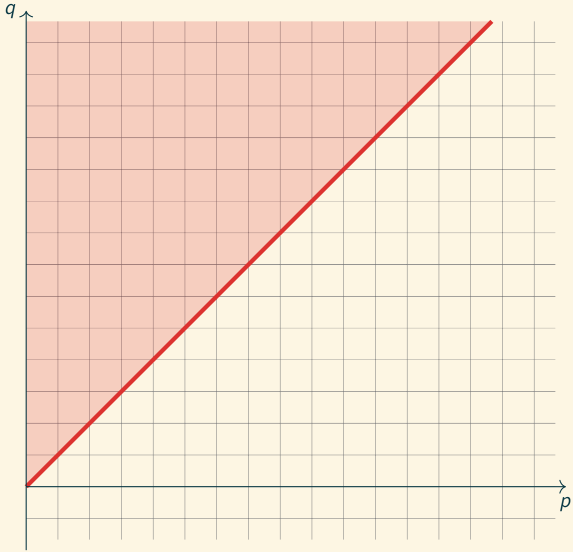

Bousfield and Kan 1972 spectral sequence of the space of minimal A_∞-algebra structures on H^*A, pointed at the minimal model of A,

$$\mathrm{Map}(\mathcal{A}_\infty, \mathcal{E}(H^*A)) = \lim_n \mathrm{Map}(\mathcal{A}_n, \mathcal{E}(H^*A));$$

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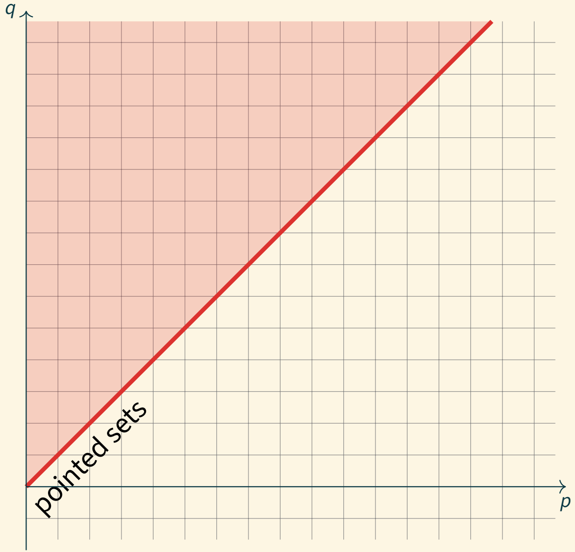

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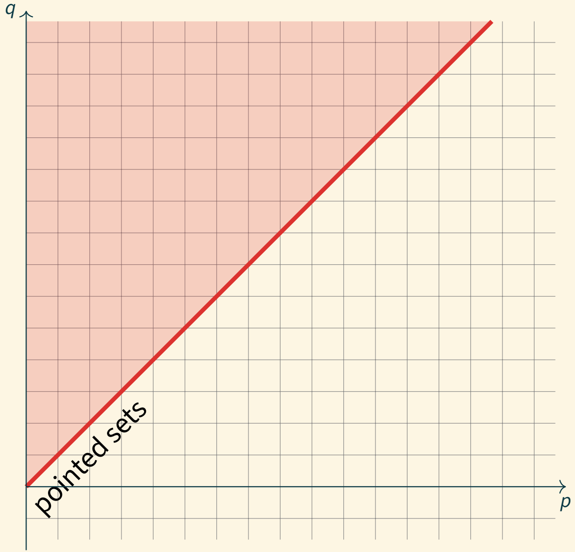

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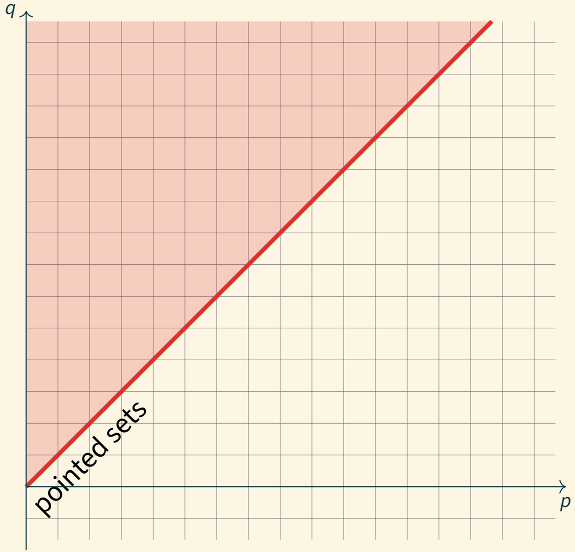

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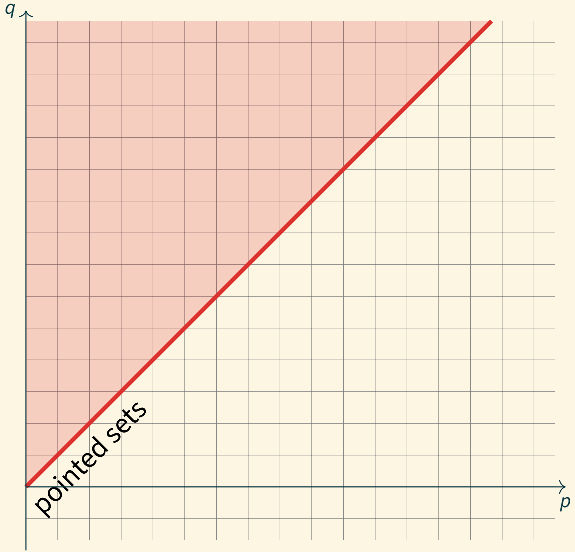
$$d_2 = 0, \qquad E_3^{pq} = E_2^{pq};$$

Proof of the generalized Kadeishvili theorem



Bousfield and Kan 1972 spectral sequence of the space of minimal A_∞-algebra structures on H^*A, pointed at the minimal model of A,

$$\mathrm{Map}(\mathcal{A}_\infty, \mathcal{E}(H^*A)) = \lim_n \mathrm{Map}(\mathcal{A}_n, \mathcal{E}(H^*A));$$

$$E_2^{pq} \Rightarrow \pi_{q-p} \mathrm{Map}(\mathcal{A}_\infty, \mathcal{E}(H^*A));$$

$$E_2^{pq} = \mathrm{HH}^{p+2,-q}(H^*A, H^*A), \quad p > 0;$$

$$d_2 = 0, \qquad E_3^{pq} = E_2^{pq};$$

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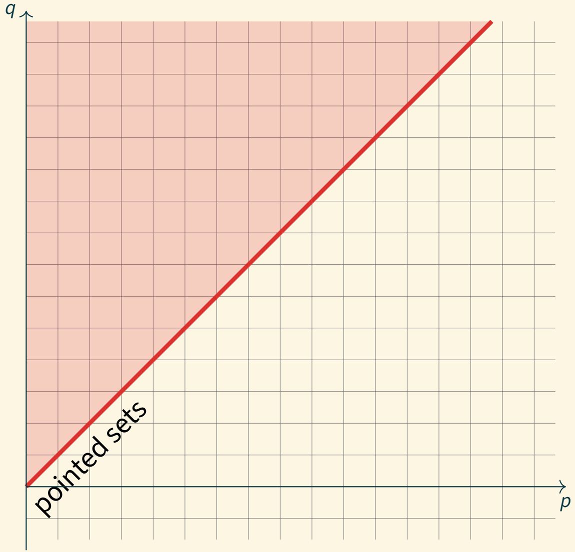

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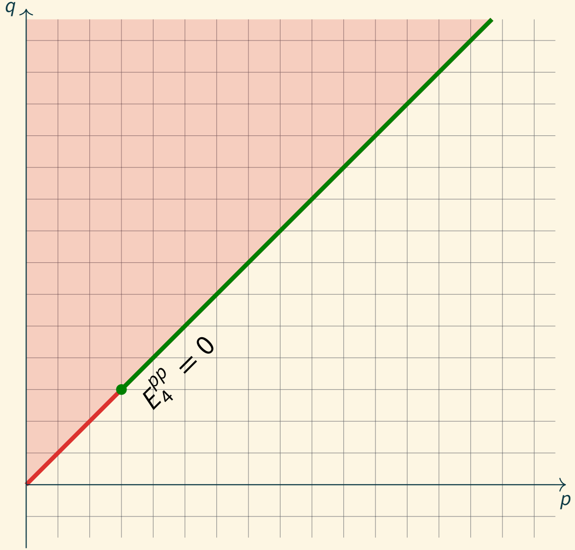
Proof of the generalized Kadeishvili theorem



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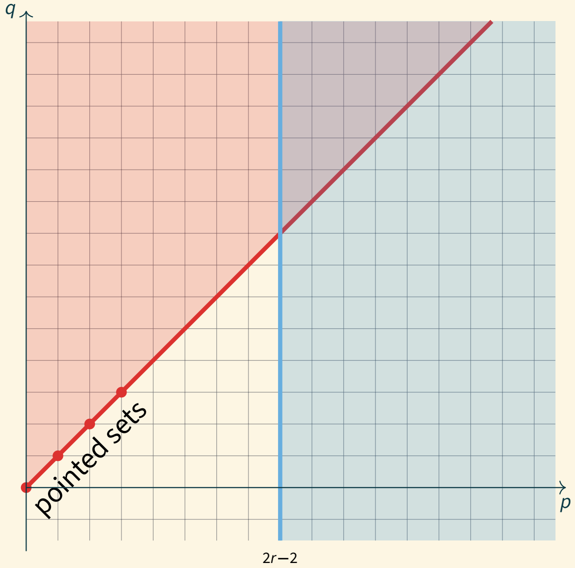

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THE END

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